Thermodynamic fluctuations in canonical shock-turbulence interaction

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ABSTRACT

In this work, we use direct numerical simulation and linear interaction analysis to study the thermodynamic field generated by the interaction of a shock wave with homogeneous isotropic turbulence. Fluctuations in density, pressure, temperature and entropy can play an important role in shock-induced mixing, combustion and energy transfer processes. Data from high-fidelity simulations is used to investigate the variation of thermodynamic fluctuations with flow Mach number for a constant turbulent Mach number and Reynolds number. As expected, density, pressure and temperature variances attain large values at the shock, followed by, in general, a decaying profile in the downstream flow. There are, however, cases with non-monotonic variation with Mach number as well as local peaks in density fluctuations behind the shock. These are explained in terms of the contribution of the post-shock acoustic and entropy modes and their cross-correlation to the thermodynamic variances. Budget of the transport equations reveal interesting insight into the physics governing the trends observed behind the shock wave. It is found that the downstream evolution of the thermodynamic field is determined by competing dilatational and dissipation mechanisms. The dominant mechanisms are identified for a range of conditions and their implication for developing predictive models is highlighted.

INTRODUCTION

Shock waves are characteristic features in compressible flows, specifically in the supersonic/hypersonic flow regime. In aerospace applications, the effect of shock waves on the turbulent features in a supersonic boundary layer is usually studied to understand the physical mechanisms responsible for boundary layer separation, increased heat transfer and high surface pressures, each of which are unique engineering problems. The interaction of free turbulence with a planar shock wave is a similar problem of interest, exhaustive with physical insights on the effects of shock on turbulence and vice-versa without additional complexities of mean shear, streamline curvature, wall effects, etc. Shock-turbulence interaction has implications in a variety of applications, to name a few, supersonic/hypersonic propulsion systems, inertial confinement fusion, shock wave lithotripsy, and astrophysical shock waves. In this study, we focus on the interaction of a homogeneous/isotropic turbulence interacting with a nominally planar shock wave, which is the most basic form of shock-turbulence interaction.

Initial studies of shock-turbulence interaction were theoretical, mostly employing linear analysis and are based on the Kovácsnay (1953) decomposition of turbulence. The theory aptly called as the linear interaction analysis (LIA) was developed by Moore (1954) and Ribner (1954) to analyze the interaction of acoustic and vorticity waves respectively with an unsteady normal shock. The theory was later put to use by many researchers to study problems involving shocks and free turbulence (e.g., Mahesh et al. (1996), Fabre et al. (2001), Wouchuk et al. (2009), Quadros et al. (2016b)). Physical experiments utilizing grid-generated turbulence interacting with a normal shock have also been carried out (cf. Agui et al. (2005) and references therein) to study the underlying mechanisms responsible for amplification of turbulent kinetic energy (TKE) and the modification of length scales across the shock; purely driven by the need to understand mixing enhancement across the shock and shock wave/turbulent boundary layer interaction (SBLI). Several numerical simulations have been carried out to study the canonical shock-turbulence interaction problem due to its geometric simplicity and rich physics (e.g., Lee et al. (1997); Larsson et al. (2013); Ryu & Livescu (2014)), with the focus of understanding the amplification of turbulent kinetic energy and entrainment, reduction in length scales, and enhanced anisotropy in post-shock Reynolds stresses.

Compressible turbulent flows are characterized by the appreciable changes in thermodynamic quantities such as density, pressure and temperature fluctuations, which can be drastically amplified/ altered on interaction with shock waves. The importance of understanding the thermodynamic fluctuations is well known as they play a major role in turbulent mass flux, sound generation, turbulent heat flux, and most importantly, in the turbulent transport of energy between internal and kinetic energy components.

The objective of the present study is to perform a detailed investigation of the thermodynamic aspects of the canonical shock-turbulence interaction. This work attempts to explain the amplification and evolution mechanisms of thermodynamic fluctuations in canonical shock-turbulence interaction by systematic variation of the governing parameters in the direct numerical simulations (DNS): flow Mach number ($M_f$), turbulent Mach number ($M_t$) and Taylor-scale Reynolds number ($Re_T$). The physical mechanisms behind the variations in thermodynamic quantities are understood using LIA and the extensive DNS data of Larsson et al. (2013). Additional numerical simulations have been carried out to supplement the existing data. A controlled study is carried out with purely vortical incoming turbulence in LIA and mostly vortical with minimal compressible fluctuations in the DNS data.
**METHODODOLOGY**

We use LIA and DNS to study the problem of a homoge-
neous/isotropic turbulence (HIT), being convected by a uniform
one-dimensional (1D) mean flow and interacting with a planar nor-
mal shock. Cartesian coordinate system is used where the shock-
normal direction is represented by $x_1$ and the shock-parallel direc-
tions by $x_2$ and $x_3$. We use the notion of Favre averages (density-
weighted) for all quantities from the numerical simulations except
for density ($\rho$) and pressure ($p$). The Favre averages are denoted
as $\bar{f}$ and the associated fluctuations are given by double primes.
In LIA, we use Reynolds averaged quantities denoted by an overbar
and their respective fluctuations denoted by single primes. When
comparing results between LIA and DNS, we make use of the assu-
mption $f \approx \bar{f}$ (linear limit). The upstream and downstream states
are denoted by subscripts ‘u’ and ‘d’ respectively.

**Linear theory**

A single vorticity wave in two-dimensions ($x_1,x_2$) of ampli-
tude $A_\psi$ and at an angle $\psi$ with the $x_1$—direction is being convected
by a 1D uniform mean flow of velocity $\bar{u}_1$ towards the normal
shock as shown in Fig. (1). The shock deforms in response and the position
of the unsteady shock is given by $\bar{\psi}(x_2,t)$. The downstream
non-acoustic waves (entropy and vorticity) are generated/refracted
at an angle $\psi$ and are convected as ‘frozen’ waves by the down-
stream mean flow with velocity $\bar{u}_d$. The acoustic waves are genera-
ted at an angle $\psi$ and are either propagating or decaying (reduction
in amplitude), depending on whether the angle of the incident wave
is lower or higher than the critical angle.

The complete details of the LIA procedure can be obtained from
Mahesh et al. (1996), Quadros et al. (2016b) and Livescu & Ryu (2016). In brief, the linearized Euler equations are used to
obtain the waveforms of the fluctuations in the upstream and down-
stream regions of the normal shock wave (modeled as a discontinu-
ity). Linearized Rankine-Hugoniot (RH) conditions are used as the
boundary conditions at the shock wave to obtain the transfer func-
tions of the post-shock fluctuations. The upstream turbulence is
represented as a super-position of 2D plane waves (Fourier modes),
with each of them independently interacting with the shock. The post-shock field thus obtained is then integrated over all of the inci-
dent waves in a specified energy spectrum (von Kármán) to obtain a
statistical description of the turbulence behind the shock wave. It is
to be noted that the upstream turbulence considered in LIA is purely
vortical with no thermodynamic fluctuations.

**Numerical simulations**

The compressible Navier-Stokes (NS) equations are solved for a
perfect gas with ratio of specific heats $\gamma = 1.4$ and zero bulk
viscosity using the solution-adaptive finite difference Hybrid code
(Larsson et al. (2013)). A linear relationship is used for relating
the viscous stresses ($\sigma_{ij}$) to the strain rates and Fourier’s law
is used for the heat flux ($q_j$). Viscosity is assumed to follow a
power-law with temperature as $\mu = \mu_{ref}(T/T_{ref})^{3/4}$. The homoge-
neous/isotropic turbulence is generated as per the method given in
Larsson et al. (2013) with the initial turbulent field specified using the
von Kármán velocity spectrum having peak energy wavenumber
of $k_0 = 4$. The isotropic databases are temporally decayed till
‘realistic’ turbulence is obtained, followed by a blending of mul-
tiple realizations to form a sufficiently long database that satisfies
statistical convergence. Taylor’s hypothesis is used to convect the
blended database as the time-dependent inflow turbulence for the
shock-turbulence interaction domain. The inflow turbulence used
in the numerical simulations is quasi-vortical with minimal amount
of thermodynamic and compressible fluctuations. The turbulence
parameters immediately upstream of the shock are: turbulent Mach
number $M_{\infty,u} = 0.15$, Taylor-scale Reynolds number of $Re_{\lambda,u} = 33$
and dissipation-scale Reynolds number $Re_{\varepsilon,u} \approx 135$.

Simulations with four different upstream Mach numbers ($M_u = 1.23, 1.50, 2.50$ and $3.50$) have been carried out in the present study.
The computational grid is stretched in the shock-normal direction ($x_1$) to facilitate finer grid sizes near the shocks. A uniform grid
spacing in used for the shock-parallel directions ($x_2$ and $x_3$). The
grid sizes are varied to ensure that the anisotropically compressed
post-shock turbulence is captured as the shock strength varies be-
tween the cases. An optimized numerical sponge is attached at the
end of the domain to gradually dissipate the spurious oscillations
from the outflow. A non-reflecting outflow boundary condition and a
back pressure controller to adjust the mean shock position are
setup according to the method provided in Larsson & Lele (2009).
The present case details are tabulated in table (1) and the simul-
ations have been validated by comparing with the available data of
Larsson et al. (2013). We note that the current simulations have
grid resolution comparable to the viscous shock thickness for the
low Mach number cases and vary by a factor of $4 - 5$ for the high
Mach number cases.

<table>
<thead>
<tr>
<th>$M_u$</th>
<th>$L_{\infty,u}/\eta_u$</th>
<th>$\Delta x_1/\Delta x_2$</th>
<th>$\delta/\eta$</th>
<th>$\text{Grid}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.23</td>
<td>45</td>
<td>0.75</td>
<td>0.90</td>
<td>882 × 384$^2$</td>
</tr>
<tr>
<td>1.50</td>
<td>45</td>
<td>0.50</td>
<td>0.41</td>
<td>1042 × 384$^2$</td>
</tr>
<tr>
<td>2.50</td>
<td>46</td>
<td>0.40</td>
<td>0.14</td>
<td>1142 × 384$^2$</td>
</tr>
<tr>
<td>3.50</td>
<td>46</td>
<td>0.33</td>
<td>0.08</td>
<td>1313 × 384$^2$</td>
</tr>
</tbody>
</table>

1. Ratio of dissipation length scale to Kolmogrov length scale - a measure of turbulence Reynolds number
2. Ratio of shock-normal grid size at the shock to that of the shock-parallel grid size - a measure of grid resolution at the shock
3. Estimate for viscous shock thickness taken from Ryu & Livescu (2014): $\delta/\eta \approx 7.69 M_{\infty,u}/(\sqrt{Re_{\lambda,u}}(M_u - 1))$
RESULTS AND DISCUSSIONS

We compare thermodynamic variances ($\bar{\rho}'^2$, $\bar{p}'^2$, $T'^2$, $\bar{e}'^2$) obtained from DNS and LIA. We discuss the spatial variation of the thermodynamic fluctuations first, followed by a discussion on the effect of upstream Mach number. Kovásznay type mode decomposition is followed to split the thermodynamic variances into their respective elementary modes as given in Quadros et al. (2016a). This enables to understand the underlying linear mechanisms in the amplification of the thermodynamic fluctuations.

We have normalized the thermodynamic variances by the square of their mean downstream values ($\bar{\rho}'^2 / \bar{\rho}'^2$, $\bar{p}'^2 / \bar{p}'^2$, $T'^2 / T'^2$, $\bar{e}'^2 / \bar{e}'^2$) except for entropy. Entropy variance is normalized by the square of the gas specific heat capacity at constant pressure ($c_v^2$). Additionally, the variances are normalized by the ratio of upstream turbulent kinetic energy to the square of the upstream mean shock-normal mean energy ($0.5 M_u^2 / \epsilon_u$) so as to remove the effect of upstream turbulence intensity. We normalize the streamwise direction by the peak energy wavenumber ($k_0$). This normalization procedure is used throughout this article, unless otherwise specified. The shock-normal direction in numerical simulations is shifted such that the mean shock location (identified as the location where the mean dilatation is most negative) is aligned with the mean shock location in LIA, i.e., centered at $k_0 x_1 = 0$.

Spatial variation

The thermodynamic variances shown in figure 2 have almost negligible values in the flow upstream of the shock wave. They exhibit a jump to relatively high values across the shock, followed by a rapid decay in the downstream flow, except entropy, which instead shows a gradual decay. In DNS, the interaction yields locally high values of the fluctuations in the shock region, which are due to the unsteady oscillation of the shock wave and do not represent turbulence.

Thermodynamic fluctuations predicted by LIA are comparable to that obtained from the DNS for low Mach numbers ($M_a < 2$), mainly due to the ‘nearly isentropic’ nature of the post-shock field for these shock strengths. At the higher Mach numbers ($M_a > 2$), there is a significant generation of entropy fluctuations. The density variance shows a non-monotonic variation behind the shock, similar to that of the shock-normal Reynolds stresses. This is also observed in LIA till the acoustic adjustment region ($k_0 x_1 \lesssim 2$), beyond which there are differences between DNS and LIA due to the viscous decay (which is not considered in the LIA framework). DNS pressure fluctuations appear to approach a finite asymptotic value in the far-field ($k_0 x_1 \rightarrow \infty$) and is comparable to that obtained from LIA. The fact that there are pressure fluctuations that are unaffected by viscous decay is entirely consistent with the notion that the shock produces a far-field radiating sound field. There is a slight mismatch in $\bar{p}'^2$ in the region immediately behind the shock (around $k_0 x_1 = 1$) indicating the effect of the non-linear mechanisms in DNS; however, the effect is minimal.

The temperature variances in DNS show a rapid decay immediately behind the shock, similar to the inviscid decay of the pressure fluctuations and a gradual decay approximately after the acoustic adjustment region ($k_0 x_1 \gtrsim 1$). LIA, on the other hand, predicts the rapid decay after the post-shock amplification but followed by a constant far-field value after the adjustment region. The post-shock entropy fluctuations as predicted by LIA match the DNS values obtained immediately behind the shock, but do not reproduce the viscous decay observed in DNS in the downstream region.

Figure 2: Spatial variation of the thermodynamic variances for $M_u = 1.5$ (red - solid lines and squares) and 3.5 (blue - dashed lines and circles). Lines represent LIA solution and symbols represent DNS results taken from present simulations. Vertical dotted and dashed lines represent the bounds of the mean shock thickness for $M_a = 1.5$ and $M_a = 3.5$ cases respectively. Note that the entropy variance generated by the $M = 1.5$ shock is scaled up by a factor of 5 to appear in the same plot.

Kovásznay modes

The Kovásznay modes of entropy and acoustics (or pressure) are fundamentally different in terms of their evolution behind the shock wave. The majority of the acoustic energy decays with distance from the shock wave; in particular, those associated with the single-wave interactions exceeding the critical angle. On the other hand, the entropy mode generated at the shock propagates downstream with no change in amplitude as per the linear inviscid framework adopted by LIA.

The LIA solution in the post-shock region is a superposition of acoustic, entropy and vorticity waves, with a collection of orientation angles and wavelengths. The density variance is thus (since the pure vorticity contribution, $\rho'_d x_1 = 0$),

$$\rho'_d \rho'_d = \left(\rho'_d a + \rho'_d e\right)^2 = \rho'_d a \rho'_d a + \rho'_d e \rho'_d e + 2 \rho'_d a \rho'_d e \tag{1}$$

which allows us to decompose the full density variance into a pure acoustic contribution (“acoustic-acoustic”, the first term), a pure entropy contribution (“entropy-entropy”, the second term), and a combined acoustic/entropy contribution (“acoustic-entropy”, the third term). A similar decomposition is done for the temperature variance.

Figure 3 shows the components of density and temperature variance for the low shock strength ($M_u = 1.5$) and the high shock strength ($M_u = 3.5$) cases. The “acoustic-acoustic” component in both the variances has large values immediately behind the shock and decay in the streamwise direction, similar to $\bar{p}'^2$. The “acoustic-entropy” correlation also shows a decaying pattern and vanishes beyond the acoustic adjustment region ($0 < k_0 x_1 \gtrsim 1$). The “entropy-entropy” component is constant with the streamwise direction, and
to the contribution of the propagating acoustic waves. That the far-field density and temperature variances are mostly due to the effect of non-linear mechanisms in that location (see figure 2 (b)). There are large variations between LIA and DNS results as the turbulent Mach number is increased, and a good match between the two when the Reynolds number is increased, which is in agreement with the assumptions of LIA.

Pressure fluctuations appear to quickly rise to a finite value even for weak interactions (\(M_u < 1.5\)) and then saturate at high Mach numbers. The post-shock entropy fluctuations appear to increase as the square of the upstream Mach number. As a result, they attain small values for weak shock interactions and rapidly increase with Mach number beyond 2. The density and temperature variances show a variation which is a combination of the acoustic and the entropy fluctuations. This is distinct in the density fluctuations (LIA data) where the variation is similar to the pressure fluctuations for \(M_u < 1.65\) and follows the trend of the entropy fluctuations for \(M_u > 2\). The DNS data (see the variation of \(M_u = 0.22\) data) also follows this trend obtained from LIA.

The near-field thermodynamic variances (obtained from LIA) are also plotted against Mach number in figure 4. Near-field density variances attain a peak value at low Mach numbers (\(M_u = 1.2\)) and reduce in amplitude as the shock strength increases, till they vanish at very high Mach numbers. Near-field values of pressure variances show a similar trend, except that they saturate to a constant value at very high Mach numbers. Temperature variances are found to be monotonically increasing as the shock strength increases in the near-field region, and have values larger than at the \(k_0x_1\) location. LIA predicts entropy fluctuations to be spatially invariant and thus the trend is observed to be the same as \(k_0x_1 = 1\) location.

**Transport of the variances**

The physical mechanisms underlying the post-shock evolution of the thermodynamic variances is next analyzed in terms of the budgets of their respective transport equations. The transport equations are obtained by multiplying the thermodynamic fluctuations with their respective transport equation, followed by a suitable averaging process. The complete transport equations for the variances can be found in Gerolymos & Vallet (2014). For completeness, the transport of pressure variance is given in equation (2),

\[
\begin{align*}
\frac{\partial}{\partial t}(\bar{p}^2) + \frac{\partial}{\partial x_j}(\bar{u}_j\bar{p}^2) &= -\frac{\gamma}{\gamma - 1} \frac{\partial}{\partial x_j}(\bar{u}_j \bar{p}^2) + 2(\gamma - 1) \frac{\partial}{\partial x_j}(\bar{p} \bar{q}_j) \\
&= -\frac{\gamma}{\gamma - 1} \frac{\partial}{\partial x_j}(\bar{u}_j \bar{p}^2) + 2(\gamma - 1) \frac{\partial}{\partial x_j}(\bar{p} \bar{q}_j)
\end{align*}
\]

The other variances also do have similar terms such as diffusion by velocity fluctuations, production due to mean gradients, correlations
with fluctuating dilatation, and triple correlations.

Figure 5 shows the budget of the terms in the transport equation of the density variance for two shock strengths, computed as part of the current work. The convection of density variance \((\overline{u \partial_{\overline{u}} \rho^2})\) is found to be balanced by the correlation of density fluctuations with the fluctuating dilatation \((\overline{\rho' \partial_{\overline{u}} u'})\). It is found that the other terms (production, diffusion, and triple correlations) do not have significant contribution in the budget. It is to be noted that transport of density variance does not have any viscous dissipation terms.

The DNS data shown in figure 6 (a) corresponds to the transport of pressure variance where, the correlation of dilatation with pressure is found to be the dominant term, similar to density variance. On the other hand, the DNS data shows that the entropy variance to be governed by dissipation mechanisms only. Figure 6 (b) shows the balancing of the convection of entropy variance by its dissipation term \((\overline{q_i/T} \partial_{\overline{u}} x_i^j)\) for all Mach numbers. Entropy variance is not affected by the acoustic mode and thus, does not have any dilatational correlations.

Transport of temperature variances is governed by dilatational correlations at all Mach numbers, according to the DNS data. Figure 7 shows the dominant terms in the budget of temperature variance, where it is found that the dissipation term has a small but finite contribution to the budget at high Mach numbers. Similar to other variances, the contribution of production, diffusion, and third order correlation terms are insignificant.

**SUMMARY**

In this work, we presented a comparison of the thermodynamic fluctuations generated by canonical shock-turbulence interaction between LIA and DNS. A large parameter space including a range of Mach number, turbulent Mach number and Reynolds number is considered, and available DNS data is complemented with new cases computed here. The linear interaction analysis based on Kovásznay mode decomposition of the disturbances is applied under the linear inviscid framework and is further used to investigate the thermodynamic field in terms of their component Kovásznay modes.

It has been known that pressure, density and temperature fluctuations are produced in a shock-turbulence interaction and they decay rapidly in the downstream flow. This is primarily associated with the acoustic mode, and reciprocates the exponential de-
cay of the acoustic energy generated at the shock. ‘Representative’ far-field values show steady increase in magnitude with increasing strength of interaction. This is essentially due to the entropy mode contributions to the density and temperature fluctuations. There are interesting non-monotonic trends, for example, in the variation of the near field $\rho^2$ and $\rho T^2$ values with Mach number. The density variance generated by strong shocks ($M \geq 2.5$) also show a non-monotonic variation (and a local peak in magnitude) with distance from the shock wave. These are caused by the interplay between acoustic and entropy modes generated by the interaction.

Transport equations are written to investigate the evolution of the thermodynamic variances downstream of the shock interaction. DNS data is used to compute the budget of the source terms for a range of Mach numbers. We identified that the transport of the thermodynamic variances in the post-shock region to be dominated by dilatational and dissipative mechanisms. The transport equations can be approximated to a suitable degree in the form of:

$$\tilde{u} \partial_{x_1} \rho^2 \approx -2\tilde{\rho} \rho \partial_{x_1} u_1'', \quad (3)$$

$$\tilde{u} \partial_{x_1} \rho T^2 \approx -2\gamma \tilde{\rho} \rho \partial_{x_1} u_1'', \quad (4)$$

$$\tilde{u} \partial_{x_1} T^\alpha \approx -(2/c_v)\tilde{\rho} T^{\alpha-1} \partial_{x_1} u_1'' - (2/c_v)\tilde{\rho} \partial_{x_1} T''', \quad (5)$$

$$\tilde{u} \partial_{x_1} \rho v^2 \approx -2(4/v)\tilde{\rho} T' \partial_{x_1} v''', \quad (6)$$

where, $c_v$ is the gas specific heat capacity at constant volume.

Dilatation effect is found to be the primary source term for the transport of $\rho^2$, $\rho T^2$ and $T''$ at low Mach numbers. Transport equations for the thermodynamic variances obtained from the linearized inviscid equations also show that the dilatation correlation to be the only source term balancing the convective term without any dissipative mechanisms. The dilatation term can be modeled effectively using the exponential decay of acoustic energy generated by the shock wave. A differential equation model to predict the post-shock variation of $\rho^2$ can be derived using LIA. A similar approach is expected to work for $\rho^2$ and $T''$ at low Mach numbers where the contribution of entropy component is small. The extensive data presented for $\rho^2$, $\rho T^2$, $T''$ will be valuable for the model validation.

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