

Physics Based Modeling of Turbulent Heat Flux in Shock Dominated Flows

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Interaction of a purely vortical isotropic turbulence with a normal shock wave is considered for analysis. Theoretical tool called Linear Inviscid Analysis (LIA) is used for studying this canonical shock turbulence problem. R_{uT} which represents the normalized turbulent heat flux, is studied in the region downstream of the shock for a range of upstream Mach numbers. DNS data available from literature for similar configuration is used as benchmark for comparison. Effect of M_t and Re_λ on generation and evolution of R_{uT} is studied from DNS data and compared with LIA results. Finally, the effect of upstream energy spectrum shape on prediction of R_{uT} is analyzed.

1. Introduction

Shock-boundary layer interaction is commonly seen in high speed flows and is an important phenomenon. In the region of such interactions, high wall pressure and wall heat flux is observed along with topological changes such as boundary layer separation [1]. Accurate prediction of these high aero-thermal loads is important from a design point of view. Numerical simulation of such flows of engineering interest largely relies on Reynolds Averaged Navier Stokes (RANS) equations.

RANS equations solve for the mean flow variables. The effect of turbulence on the mean flow is modeled in terms of suitable constants and mean flow variable gradients. Conventional RANS models incorrectly predict the wall heat flux in the region of shock-boundary layer interaction [2]. This indicates to additional physics that is uncaptured by RANS models and is a cause for further investigation.

An important term in the Reynolds averaged energy equation is the turbulent heat flux vector $\overline{u'h'}$ where u' and h' denotes the velocity and enthalpy fluctuations respectively. This term represents the turbulent convection of fluctuating internal energy or enthalpy by velocity fluctuations. This correlation is modeled as per gradient diffusion hypothesis by considering it proportional to mean temperature gradient. The proportionality constant is written in terms of eddy viscosity μ_t and turbulent Prandtl number Pr_t [3].

Morkovin hypothesis applicable for compressible boundary layer suggests that the velocity-temperature correlation coefficient $R_{uT} = -1$ which implies a Pr_t value of 1 [4]. RANS models consider a constant value of $Pr_t = 0.89$ for modeling turbulent heat flux in boundary layer flows. It is to be noted that the basis for Morkovin hypothesis relies on the fact that the total temperature fluctuations are small in the flow field. The upstream turbulent fluctuations interacting with the shock, makes it oscillate about a mean position and the total temperature fluctuation across such a shock wave is of significant level [5].

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This indicates to a varying Pr_t value across the shock and in the past varying Pr_t RANS models have shown improved heat flux results in shock dominated flows [2].

The objective in this work is to study the generation of turbulent heat flux correlation as the flow passes through the shock wave. For this study, we consider the canonical form of homogeneous isotropic turbulence interacting with a normal shock. The turbulence is purely vortical with no temperature, pressure or density fluctuations and is carried by a 1-D uniform mean flow. This isolates the effect of turbulence on the shock without getting in additional complexities such as boundary layer gradient, streamline curvature, flow separation etc. Extensive DNS data [6] is also available for this simplified case thus providing significant insight into essential physics of the problem. A theoretical tool called Linear Inviscid Analysis (LIA) is used for studying this problem.

LIA models the upstream turbulence as combination of 2-D planar disturbance waves [7]. Each of these waves is considered to interact independently with the shock, generating disturbance that could be vortical, acoustic and entropic in nature. The wave properties behind the shock are obtained by solving linear Euler equations in the downstream regime with linear Rankine-Hugoniot equations applied at the shock boundary. The overall downstream turbulence is a linear superposition of the results obtained for these independent interactions [5].

Using LIA we study the generation of streamwise turbulent heat flux $\overline{u'T'}$ across a shock. The DNS data is available for cases with upstream flow having different mean Mach number, turbulent Mach number and Reynolds number upstream of the shock. Correlation coefficient R_{uT} obtained downstream of the shock is studied for these cases of varying M_t and Re_λ and compared with LIA. Finally, the effect of upstream turbulence spectrum on the generation of post-shock correlation coefficient is also studied.

2. Flow description

A steady, 1D, uniform mean flow is considered upstream and downstream of a normal shock. The mean flow upstream of the shock carries a purely vortical turbulence which is homogeneous and isotropic in nature. The shock distorts from its mean position due to the presence of upstream turbulence. The shock in turn amplifies the turbulence as it passes through it.

For the purpose of theoretical analysis, the shock distortion from its mean position is described by a distance $\xi(y, z, t)$ (see figure 1). The temporal derivative of shock deviation given by ξ_t represents the stream wise velocity of the shock wave. The transverse derivative ξ_y and ξ_z denote the angular deviation of the shock in $x - y$ and $x - z$ planes respectively where x , y and z represent the coordinates in the lab frame of reference. The associated velocities are u , v and w respectively.

3. Methodology

3.1. LIA formulation

Consider a two-dimensional plane vortical wave of the form given below

$$\frac{u'_1}{U_1} = lA_v e^{ik(mx+ly-U_1mt)} \quad (3.1)$$

$$\frac{v'_1}{U_1} = -mA_v e^{ik(mx+ly-U_1mt)} \quad (3.2)$$

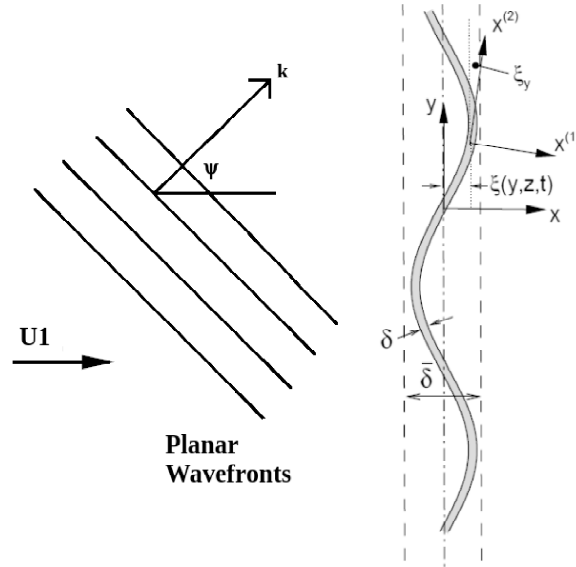


FIGURE 1. Schematic showing a shock wave distorted upon interaction with turbulent fluctuations

$$p'_1 = T'_1 = \rho'_1 = 0 \quad (3.3)$$

u' and v' representing fluctuations in the streamwise and transverse directions. p' , T' and ρ' represent the fluctuations in pressure, temperature and density respectively. Also, $m = \cos \psi$ and $l = \sin \psi$ where ψ is the angle between wave number vector and the mean flow direction as shown in figure 1. A_v and k represent the complex amplitude and the wave number of upstream vorticity wave.

Upon interaction of this single vorticity wave with the shock, three waves are generated. Those are acoustic, vorticity and entropy waves. Acoustic wave has its contribution to pressure, velocity, temperature and density. Entropy wave has its contribution to temperature and density. Vorticity wave has its contribution only to velocity. Therefore, the waveforms downstream of the shock can be written as

$$\frac{u'_2}{U_1} = F e^{i\tilde{k}x} e^{ik(ly-mU_1t)} + G e^{ik(mrx+ly-mU_1t)} \quad (3.4)$$

$$\frac{v'_2}{U_1} = H e^{i\tilde{k}x} e^{ik(ly-mU_1t)} + I e^{ik(mrx+ly-mU_1t)} \quad (3.5)$$

$$\frac{T'_2}{T_2} = \frac{\gamma-1}{\gamma} K e^{i\tilde{k}x} e^{ik(ly-mU_1t)} + Q e^{ik(mrx+ly-mU_1t)} \quad (3.6)$$

$$\frac{\rho'_2}{\bar{\rho}_2} = \frac{K}{\gamma} e^{i\tilde{k}x} e^{ik(ly-mU_1t)} + Q e^{ik(mrx+ly-mU_1t)} \quad (3.7)$$

$$\frac{p'_2}{\bar{p}_2} = K e^{i\tilde{k}x} e^{ik(ly-mU_1t)} \quad (3.8)$$

where F , H and K represent the magnitude of acoustic components related to stream-wise velocity fluctuation, stream-normal velocity fluctuation and pressure fluctuation. G and I represent the contribution from vorticity wave to stream-wise velocity fluctuation and stream-normal velocity fluctuation. Q represents the contribution of entropy wave to both temperature and density fluctuation. Also, $\tilde{k}$ represents wave number of the generated acoustic wave. Streamwise wave number for the downstream vorticity and entropy wave is amplified by a factor r which denotes the density ratio, $\bar{\rho}_2/\bar{\rho}_1$.

The wave characteristics associated with shock are given by

$$\frac{\xi_t}{U_1} = L e^{ik(l_y - m U_1 t)} \quad (3.9)$$

$$\xi_y = -\frac{l}{m} L e^{ik(l_y - m U_1 t)} \quad (3.10)$$

where ξ_t and ξ_y represent shock speed and shock distortion. The boundary condition at the shock is given by linearised Rankine-Hugoniot(R-H) equations.

$$\frac{u'_2 - \xi_t}{U_1} = B_1 \left(\frac{u'_1 - \xi_t}{U_1} \right) \quad (3.11)$$

$$\frac{v'_2}{U_1} = \frac{v'_1}{U_1} + E_1 \xi_y \quad (3.12)$$

$$\frac{\rho'_2}{\bar{\rho}_2} = C_1 \left(\frac{u'_1 - \xi_t}{U_1} \right) \quad (3.13)$$

$$\frac{p'_2}{\bar{p}_2} = D_1 \left(\frac{u'_1 - \xi_t}{U_1} \right) \quad (3.14)$$

B_1 , C_1 , D_1 and E_1 are functions of upstream Mach number given by

$$B_1 = \frac{(\gamma - 1)M_1^2 - 2}{(\gamma + 1)M_1^2}$$

$$C_1 = \frac{4}{(\gamma - 1)M_1^2 + 2}$$

$$D_1 = \frac{4\gamma M_1^2}{2\gamma M_1^2 - (\gamma - 1)}$$

$$E_1 = \frac{2(M_1^2 - 1)}{(\gamma + 1)M_1^2}$$

The governing equations downstream of the shock are linearised Euler equations.

In order to solve for the complex amplitudes, F , H , K , G , I , Q and L , we substitute the downstream waveforms, (3.4 – 3.8) in linearised R-H and Euler equations to get a set of linear algebraic equations which can be easily solved. The solutions are

$$\tilde{L} = \frac{-m - \beta D_1 l + \frac{mr}{l} [-\alpha D_1 l + B_1 l]}{E_1 \frac{l}{m} - \beta D_1 - \frac{mr}{l} (1 - B_1 + \alpha D_1)} \quad (3.15)$$

$$\tilde{K} = D_1(1 - \tilde{L}) \quad (3.16)$$

$$\tilde{F} = \alpha \tilde{K} \quad (3.17)$$

$$\tilde{H} = \beta \tilde{K} \quad (3.18)$$

$$\tilde{G} = \tilde{L}(1 - B_1 + \alpha D_1) - \alpha D_1 l + B_1 l \quad (3.19)$$

$$\tilde{I} = -\frac{mr}{l} \tilde{G} \quad (3.20)$$

$$\tilde{Q} = -\frac{\tilde{K}}{\gamma} + C_1(l - \tilde{L}) \quad (3.21)$$

The tilde on the complex variables represents normalization by A_v . For example, $\tilde{L} = \frac{L}{A_v}$. α and β are given by

$$\alpha = \frac{a_2^2}{\gamma U_1^2} \frac{\frac{\tilde{k}}{k}}{m - \frac{\tilde{k}}{kr}}$$

$$\beta = \frac{a_2^2}{\gamma U_1^2} \frac{l}{m - \frac{\tilde{k}}{kr}}$$

Thus, for a single vorticity wave upstream of the shock having wave number k at an angle ψ to the streamwise direction, the downstream properties can be calculated using (3.4)-(3.8) with the aid of solutions (3.15)-(3.21).

Turbulent statistics being an important measure of any turbulent field, can be derived from downstream waveforms. For example, the statistics associated with streamwise velocity fluctuation downstream of the shock can be calculated as

$$\overline{u_2'^2} = \overline{u_2' u_2'^*} \quad (3.22)$$

where over-line implies averaging on transverse direction and time. * denotes complex conjugate. Using (3.4) we get

$$\frac{(\overline{u_2'^2})_{2D}}{U_1^2} = [| \tilde{F} |^2 e^{i(\tilde{k} - \tilde{k}^*)x} + | \tilde{G} |^2 + \tilde{F} \tilde{G}^* e^{i(\tilde{k} - kmr)x} + \tilde{F}^* \tilde{G} e^{-i(\tilde{k}^* - kmr)x}] | A_v |^2 \quad (3.23)$$

which is the streamwise component of turbulent kinetic energy. The subscript '2D' indicates the value for a single wave analysis. The downstream turbulent energy flux $\overline{u_2' T_2'}$ can also be written in a similar way as

$$\left(\overline{u_2' T_2'} \right)_{2D} = 0.5 \left(\overline{u_2' T_2'^*} + \overline{u_2'^* T_2'} \right) \quad (3.24)$$

The values of the above expression can be calculated using (3.4) and (3.6).

The upstream turbulence is modeled as a collection of waves with a 3D energy spectrum tensor

$$E(k) \sim \left(\frac{k}{k_o}\right) e^{-2\left(\frac{k}{k_o}\right)^2} \quad (3.25)$$

where k is the upstream wave number and k_o is the peak wave number. It can be further shown from analysis that $|A_v|^2$ can be written as

$$|A_v|^2 = \frac{E(k)}{4\pi k^2}$$

For a given upstream spectrum of waves, the total turbulent statistics downstream can be calculated by integrating single wave analysis results over all wave numbers and angles of incidence. Therefore, we obtain

$$\left(\overline{u'_2 T'_2}\right)_{3D} = 4\pi \int_{k=0}^{\infty} \int_{\psi=0}^{\frac{\pi}{2}} \left(\overline{u'_2 T'_2}\right)_{2D} k^2 \sin \psi \, d\psi \, dk$$

3.2. DNS data

Larsson et al. [6] carried out DNS for the case of purely vortical turbulence interacting with a normal shock. Simulations were performed for 20 cases, each varying in either upstream Mach number, turbulent Mach number or Reynolds number. Upstream Mach number considered ranged from 1.06 to 6. Turbulent Mach number for these cases were between 0.15 and 0.38. Further, two cases of upstream Reynolds number based on Taylor-scale, have been considered for study. These cases highlight the effect of shock strength, upstream turbulent intensity and viscous mechanisms on the shock-turbulence interaction.

Table 1 displays the DNS cases whose data sets have been used for the current study. Number of grid points used for each of the cases are also highlighted. For example, for the case of $M = 1.05$, the grid size is 828×384^2 where the domain consists of 828 points in x-direction and 384 points in y and z directions each. For the case of $Re_\lambda = 75$, higher number of grid points have been used as the scales of turbulence are smaller and the domain needs to be better resolved. The time dependent solution obtained from the DNS is averaged over a finite time interval to give turbulent statistics for each of these cases. The number of time steps over which the averaging is performed is displayed for individual cases.

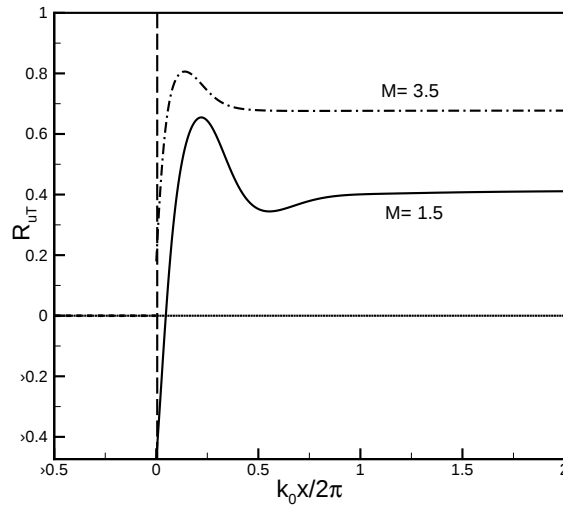
A quantity of interest with respect to DNS data is the dissipation length scale denoted by L_ϵ . The length scale of turbulence at which dissipation of turbulent kinetic energy occurs is given by

$$L_\epsilon = (R_{kk}/2)^{3/2}/\epsilon$$

where R_{kk} and ϵ represent twice of turbulent kinetic energy and dissipation rate respectively. This scale is adopted for x-directional normalization while plotting quantities of interest along the domain length x . In the present theoretical study, the peak upstream wavenumber for the incoming turbulence is taken as $k_o = 4$. x can be normalized by dividing with the wavelength of the peak wavenumber ($\lambda_o = 2\pi/k_o$). Also, $k_o \times L_\epsilon$ is a quantity that relies on the shape of the spectrum. For theoretical analysis, a value of $k_o \times L_\epsilon \sim 3-4$ can be considered. From this expression also, an equivalent L_ϵ for theory can be obtained to normalize along x-direction.

M	M_t	Re_λ	Grid Size	Time-steps considered for averaging
1.05	0.05	40	828×384^2	128
1.28	0.15	40	1040×384^2	64
1.50	0.15	40	1040×384^2	8
1.87	0.22	40	1257×384^2	8
2.50	0.22	40	1257×384^2	8
3.50	0.16	40	1257×384^2	8
4.70	0.23	40	1257×384^2	8
6.00	0.23	40	1257×384^2	8
1.50	0.14	75	2234×1024^2	48
1.50	0.22	75	2234×1024^2	48
1.50	0.38	75	2366×1024^2	48
3.50	0.15	75	2234×1024^2	48

TABLE 1. List of DNS cases used in the present study

FIGURE 2. Variation of R_{uT} with downstream distance x obtained from linear interaction analysis

4. Results

For a purely vortical upstream turbulence, a finite correlation between velocity and temperature fluctuations is obtained downstream of the shock. The velocity temperature correlation namely R_{uT} is given by

$$R_{uT} = \frac{\overline{u'T'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{T'^2}}}$$

The evolution of R_{uT} as per LIA is shown for two upstream Mach numbers in figure 2. $x = 0$ represents the shock location.

For $M = 1.5$, just downstream of the shock, R_{uT} takes a negative value and rises

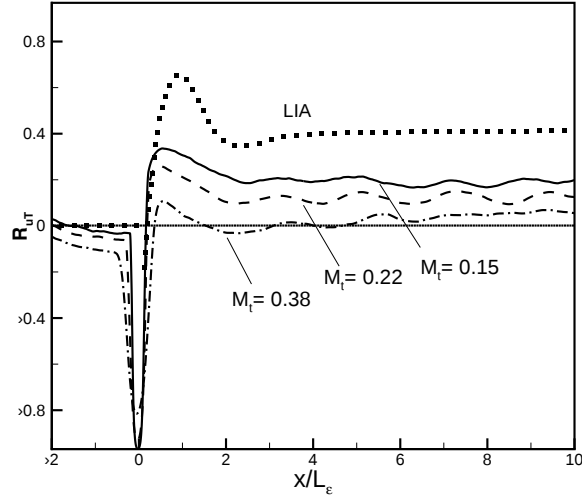


FIGURE 3. Comparison between LIA and DNS for R_{uT} evolution behind the shock; Upstream Mach number $M = 1.5$

to have a positive transient peak. Further, in the far-field, a steady positive R_{uT} value is obtained. Similar trend is seen for $M = 3.5$ case except that the value just downstream of the shock is positive. Acoustic fluctuations are generated behind the shock and they decay with distance from the shock. The transient nature of R_{uT} variation can be attributed to this phenomenon as velocity and temperature fluctuations both have acoustic components associated with them. For a higher Mach number, a higher value of far-field correlation coefficient is obtained.

Figure 3 also shows generation of R_{uT} in the region downstream of the shock. DNS and LIA results are compared for an upstream Mach number $M = 1.5$. Three different upstream M_t values are taken from the available DNS data sets for comparison. Similar to linear theory results, DNS also shows a negative near-field R_{uT} just downstream of the shock followed by a transient peak and a fairly constant far-field value. As the upstream M_t is decreased, the R_{uT} values head closer towards LIA result. This corroborates with the fact that LIA is applicable for fluctuations of small magnitude as compared to corresponding mean variables.

We note that $R_{uT} = 0$ in the linear theory results which have purely vortical disturbance upstream of the shock. The upstream flow in DNS is meant to have only vortical fluctuations. Due to acoustics that are unphysical, there also exists temperature fluctuations yielding a finite negative R_{uT} upstream of the shock (see figure 3). This happens especially for higher M_t values and influences the downstream R_{uT} predictions.

Figure 4 shows the variation of R_{uT} along x for Mach number $M = 1.5$ at two different M_t values. As seen, for the case of $M_t = 0.22$, the R_{uT} values post the transient peak attain a fairly constant mean value displaying some oscillations along the downstream. However, for the case of $M_t = 0.38$, it can be seen that although a sinusoidal pattern is obtained post the transient peak, the trend rises steadily along x . The reason for this steady increase in the oscillation is unknown and needs further probing.

Based on this analysis, a single far-field R_{uT} value cannot be assigned for each case. A mean value can be obtained by averaging all R_{uT} values between $x/L_\epsilon \sim 2$ and the

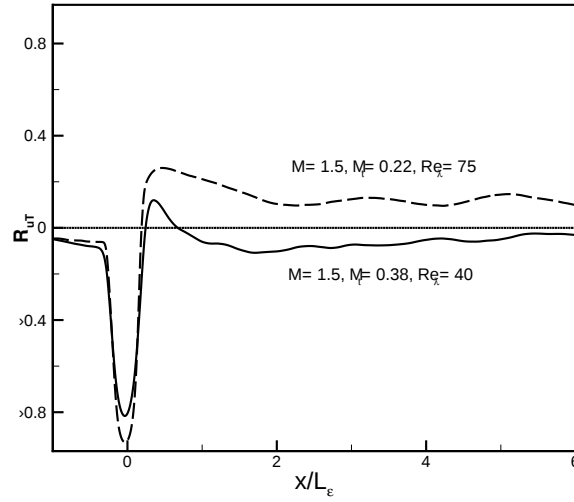


FIGURE 4. Variation of R_{uT} with downstream distance x obtained from DNS showing the difference in far-field R_{uT} evolution for different cases

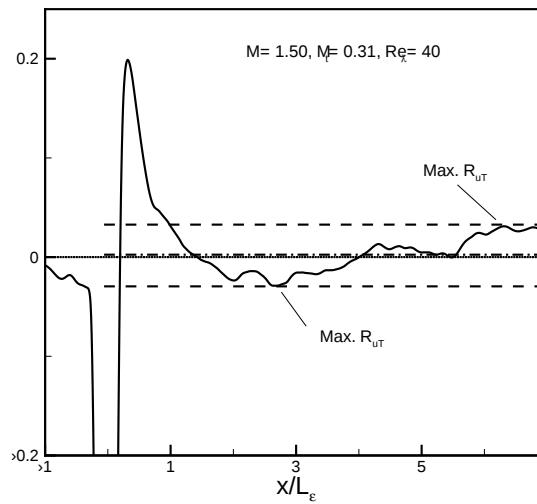


FIGURE 5. Variation of R_{uT} with downstream distance x obtained from DNS for a single case showing the maximum, minimum and average R_{uT}

end of domain. The maximum and minimum R_{uT} in this span are considered in defining the error bar. Figure 5 shows one such case where maximum, minimum and the average value of R_{uT} have been displayed.

Figure 6 shows the LIA result for near-field and far-field R_{uT} for varying upstream Mach numbers. The near-field R_{uT} for low upstream Mach number takes negative values and asymptotes to a steady positive value for $M \rightarrow \infty$. The far-field R_{uT} also displays a similar trend. It remains negative only for a very small range of upstream Mach numbers close to unity. For higher Mach numbers, the R_{uT} value is close to 0.7.

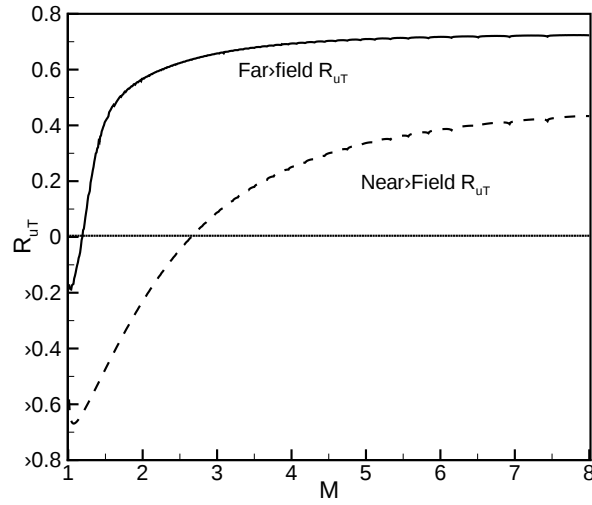


FIGURE 6. Variation of R_{uT} with upstream Mach number M for near-field and far-field obtained from LIA

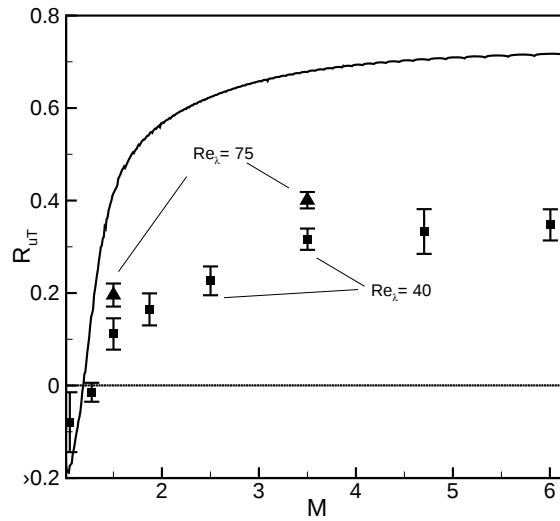


FIGURE 7. Comparison between LIA and DNS results for far-field R_{uT} for varying upstream Mach number M

From the figure, the far-field value of R_{uT} is higher than the near-field value for all upstream Mach numbers. This increase along x-direction in the post-shock region can be attributed to the decay of acoustic fluctuations generated behind the shock.

Figure 7 shows the variation of far-field R_{uT} with upstream Mach number. DNS and LIA results are compared and the effect of upstream Reynolds number on R_{uT} is shown. R_{uT} as predicted by DNS also shows negative value for very low upstream Mach numbers and reaches as steady positive value for higher Mach numbers. The DNS study considers two upstream Reynolds numbers based on Taylor micro-scale namely

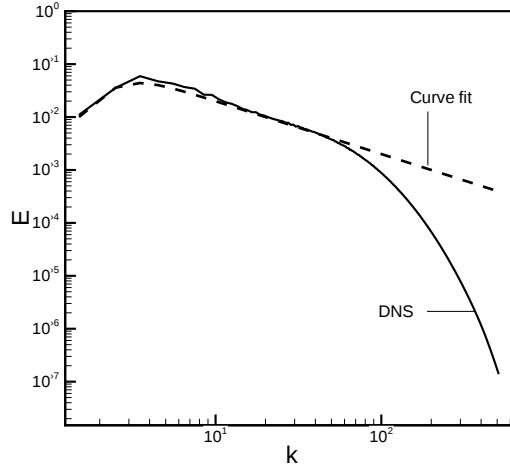


FIGURE 8. Energy spectrum obtained from DNS for the case of $Re_\lambda = 75$ and its curve-fit

$Re_\lambda = 40$ and $Re_\lambda = 75$. For the same upstream Mach number, R_{uT} corresponding to $Re_\lambda = 75$ yields a higher positive value as compared to $Re_\lambda = 40$. LIA gives a much higher value of R_{uT} as compared to DNS. It is to be noted that LIA is based on inviscid theory and formulations are for limiting case of $Re \rightarrow \infty$ ignoring the effect of viscosity.

The form of energy spectrum in the LIA formulation to represent the upstream isotropic turbulence is seen in (3.25). The spectrum however is appropriate for low Reynolds numbers. In the recent DNS simulations, the energy spectrum obtained for the case of $Re_\lambda = 75$ is shown in figure 8.

The curve-fit for this spectrum has the form

$$E(k) \sim \frac{k^4}{(k^4 + 35)^{1.25}}$$

Figure 9 shows variation of R_{uT} along x direction for a single upstream Mach number of $M = 1.5$.

Both spectra are implemented in the theory and their effect on R_{uT} generation downstream of the shock is shown. Also shown is the R_{uT} variation as per DNS for a case of low upstream M_t value. The modified spectrum reveals an early decay of acoustic fluctuations as compared to conventional spectrum result and the DNS case. However, the transient peak and the far-field R_{uT} for both the cases of LIA have similar values. This shows that for theoretical analysis, spectrum shape has no effect on the far-field R_{uT} value but only on the nature of acoustic generation behind the shock.

5. Conclusions

Generation of turbulent heat flux across a normal shock was studied. The turbulence upstream of the shock was considered to be purely vortical. For low upstream Mach numbers, LIA predicted negative R_{uT} just behind the shock. However, the value of R_{uT} reaches a steady positive value after passing through a transient peak value. For high

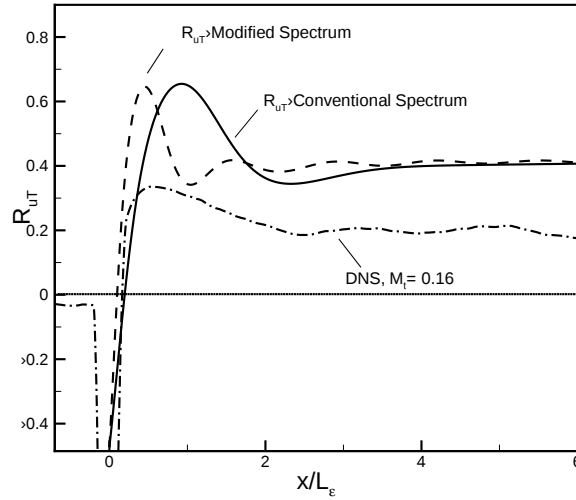


FIGURE 9. Figure shows the effect of upstream spectrum on variation of R_{uT} along x direction. The upstream Mach number is $M = 1.5$.

Mach numbers, LIA predicted similar trend except that the post-shock value of R_{uT} was positive.

A fairly good qualitative match was obtained on comparison with DNS. However LIA over-predicted the R_{uT} value for the entire downstream region. Analysis of appropriate cases of DNS showed that R_{uT} values were sensitive to the upstream turbulent Mach number M_t and Taylor-scale Reynolds number Re_λ . For low M_t values and higher Re_λ values, the DNS values approached towards theoretical predictions.

Turbulent heat flux vector modeling in RANS frame work is based on the assumption that $R_{uT} \sim -1$. A positive value of R_{uT} in the far-field indicated towards exploring alternate RANS modeling strategies for similar flows. It was also seen from theory that change in upstream energy spectrum shape, only altered the acoustic generation downstream of the shock and had no effect on far-field R_{uT} values.

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