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Numerical investigation of axisymmetric underexpanded supersonic jets

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ABSTRACT

The underexpanded supersonic jets exhausting from a rocket nozzle operating at high altitudes can have deleterious effects on the rocket's performance, e.g., reverse flows due to multi-jet interactions causing severe base heating of the jet engine. The instances at which two adjacent jets interact with each other can be predicted with a proper understanding of the individual jet flow-fields. In this work, we compute the supersonic underexpanded jets exhausted into still air and supersonic streams. The computations are performed assuming an inviscid and axially symmetric flow using an in-house RANS solver. Three sets of parametric studies have been carried out, and the parameters considered are the nozzle pressure ratio (NPR), nozzle exit Mach number, and the Mach number of the ambient air. For the first two set of parametric studies, the underexpanded jet is exhausted into still air. It is seen that as the NPR increases, keeping other parameters constant, the jet diameter and the first shock-cell length increases. The increase in jet exit Mach number for the same NPR decreases the initial jet inclination angle, and the length of the first shock-cell length increases. As the Mach number of the ambient air is increased, additional shock waves appear in the flow-field, and the change in external flow variables affect the internal structure of the jet as well.

1. INTRODUCTION

Many practical engineering applications involve under-expandedt supersonic jets, such as the exhaust and plumes of rockets. An underexpanded jet occurs whenever the nozzle exit jet pressure is more than the ambient pressure, which is the case with rockets at high altitudes. The structure of these jets has been associated with many problems such as reverse flow towards the base in rocket nozzle boosters [1] and jet noise [2].

The numerical studies and experiments conducted in ground testing facilities ([1], [3], [4], [5], [6]) indicate that the structure of the underexpanded jet depends on the parameters such as the nozzle pressure ratio (NPR), which is the ratio of static nozzle exit pressure to the ambient pressure, Mach number at the nozzle exit M_N, ratio of the specific heats, divergence angle of the nozzle, and the external ambient conditions into which the jet is exhausted. An understanding of the jet structure and its variation due to the change in these parameters help in understanding more complex flow situations such as multiple jet interactions.

A common example of this is the interaction of underexpanded jets with each other at high altitudes exhausted from the multiple strap-on boosters used in rockets for thrust generation. The interaction of jets in such situations can cause reverse flow which has detrimental effects, such as a change in the drag/thrust, severe base heating of the nozzle, etc. to name a few. Figure **12**, in Desikan *et al.* [1], shows a schlieren image of twin underexpanded jets at 3.05 Mach number with NPR = 30, coming out from two boosters of a rocket nozzle. Each nozzle has a diameter of 30 mm and the center-to-center distance between them is 80 mm. The main flow features associated with an underexpanded jet at high NPRs, such as the expansion fan, incident shock, Mach disk, triple point, reflected shock and the jet boundary are clearly visible. Depending on the jet parameters, the jets can interact with each other causing a more complex

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flow-field. In order to predict the instances of their interactions, a clear and well understanding of the behavior of the individual jets with the variation in the jet parameters is important.

In this work, we conduct parametric studies on a single underexpanded supersonic jet with the help of numerical simulations. The parameters considered are the nozzle pressure ratio (NPR), jet Mach number M_N, and the ambient Mach number. The main objective here is to study the internal structure of the jet with the variation in these parameters.

2. GENERAL DESCRIPTION OF THE FLOW FIELD

Figure **1,** adapted from Pack [7], shows the structure of an underexpanded axisymmetric supersonic jet exhausted into the still air, and the interplay between the Mach waves and compression waves is observed. At the lips of the nozzle exit, Prandtl-Meyer expansion fans are formed, which are shown in terms of the dashed lines representing the Mach waves. These Prandtl-Meyer expansion fans expand the fluid downstream of the nozzle exit to ambient conditions. There is a discontinuity in velocity across the jet boundary, however, pressure remains constant. The jet boundary bents towards the jet centerline to maintain a constant pressure along it. In order to turn the flow parallel to this boundary, the Mach waves intersect the jet boundary and reflect back to the jet centerline as compression waves. These compression waves coalesce to form a shock wave, known as the incident or intercepting shock. The incident shock can be an oblique shock or a barrel-shaped shock, depending on the degree of under expansion of the jet. Also, depending on the NPR values, the incident shock may undergo a regular reflection at the jet centerline or can result in a Mach reflection. A regular reflection leads to the formation of a reflected shock, whereas, the Mach reflection leads to the apparition of a normal shock, called the Mach disk.

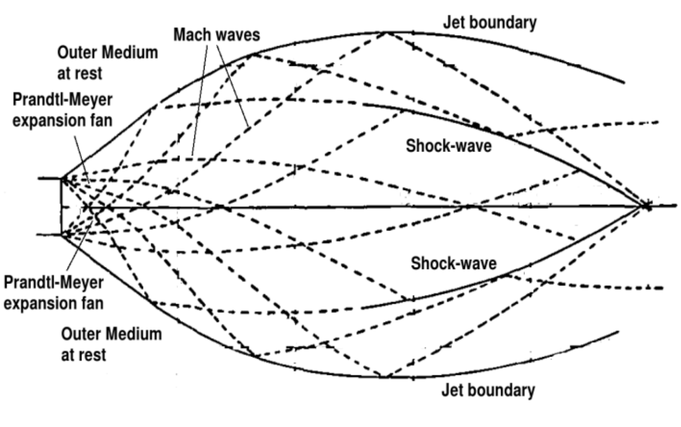

Figure 1: A diagram of an underexpanded axisymmetric supersonic jet exhausted into still air showing the interplay between the expansion and compression waves (Adapted from [7]).

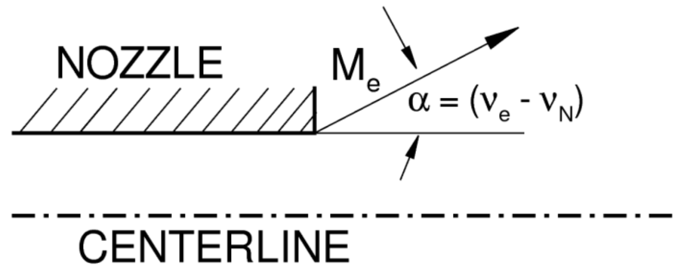

Figure 2: Initial inclination angle (α) of the jet boundary with respect to the jet centerline, as a function of the Prandtl-Meyer turning angles.

2.1 INITIAL INCLINATION OF THE JET BOUNDARY

The calculation of the initial inclination of the jet boundary is important as it predicts the condition at which the multiple jets coming out from adjacent rocket nozzle boosters will interact. An approximate calculation of the initial inclination of the jet boundary can be done based on the principles of gas dynamics. Figure **2** shows the schematic of the initial inclination of the jet boundary with the jet centerline in terms of the Prandtl-Meyer turning angles υ_e and υ_N, where the Prandtl-Meyer function $\upsilon(M)$ is given by,

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right) - \tan^{-1}\sqrt{M^2-1}$$

The flow at Mach number M_N with the corresponding Prandtl-Meyer angle υ_N expands due to the formation of expansion fan at the nozzle lip. The flow after expanding to the ambient conditions has a Mach number M_e and υ_e is the corresponding Prandtl-Meyer angle. Therefore, the total turning angle of the flow or the initial inclination of the jet boundary with respect to the jet centerline is given by, $\alpha = \upsilon_e - \upsilon_N$.

3. UNDEREXPANDED JET CONDITIONS

Table **1** lists the test cases used in this work to compute the underexpanded supersonic jet exhausting into the still air as well as to the supersonic stream. The first three cases are taken from Fu *et al.* [4], and the remaining cases are constructed to study the effects of different parameters on the jet structure. For the cases with different NPR, a constant nozzle exit condition is prescribed, and the effect of variation of NPR is brought about by varying the ambient pressure. For studying the effect of jet exit Mach number on the structure of underexpanded jet ejected into still air, a constant NPR of 17.2 is considered. Three different jet exit Mach numbers 2, 3.13, and 4.5 are used here. Finally, the cases with the jet exhausted into a supersonic stream

Table 1: Conditions of supersonic underexpanded jet exhausted into still air and supersonic streams.

Sr. No.	Jet Pressure (Pa)	Jet temp. (K)	Jet Mach number M_N	Ambient pressure (Pa)	Ambient temp. (K)	Ambient air Mach number M_∞	NPR
1	187350	1415.1	3.13	43520	282	0	4.3
2	187350	1415.1	3.13	21760	244	0	8.6
3	187350	1415.1	3.13	10880	216.7	0	17.2
4	187350	1415.1	3.13	3747	216.7	0	50
5	187350	1415.1	3.13	1873.5	222.3	0	100
6	187350	1415.1	2	10880	216.7	0	17.2
7	187350	1415.1	4.5	10880	216.7	0	17.2
8	187350	1415.1	3.13	10880	216.7	2	17.2
9	187350	1415.1	3.13	10880	216.7	4	17.2

are studied. For this, two different supersonic Mach numbers of 2 and 4 are considered with an NPR of 17.2, and jet exit Mach number of 3.13. For all the cases considered in this work, the ratio of specific heats is taken be equal to 1.4.

4. SIMULATION METHODOLOGY

4.1 GOVERNING EQUATIONS AND NUMERICAL METHOD

The computations are performed assuming inviscid and axially symmetric flow using an in-house Reynolds-averaged Navier-Stokes solver for compressible flows. The solver is based on finite volume formulation, and uses the low dissipation modified Steger-Warming flux vector splitting method to capture very thin and well-defined shock waves. First order accurate simulations are performed with implicit time integration based on the data parallel line-relaxation method to reach the steady state solution. The solver is well validated for the propulsion flow fields in high-speed aerospace applications (Sinha, Rane and Pawar [8]; Sinha and Pawar [9]; Sinha and Vadivelan [10]). For all the computations, the working fluid is taken to be air, and high-temperature effects are neglected.

4.2 COMPUTATIONAL DOMAIN AND BOUNDARY CONDITIONS

A jet nozzle exit diameter of 60 mm is used for all the computations in this work. The computational domain along with the grid structure and details of the boundary conditions are shown in Fig. **3**. The computational domain consists of a rectangular region extending to a height of 10 nozzle diameters radially from the jet centerline, and a length of 35 nozzle diameters downstream from the jet exit. For all the cases considered, the numerical simulation begins at the exit plane of the nozzle. The conditions for the jet and the ambient air presented in Table **1** are used as the inlet conditions. A hyperbolic tangent function is used to smoothen the flow gradients between the nozzle and the freestream. This type of smoothing is shown to be required to obtain a grid independent solution ([11], [12]). At the centerline of the jet, symmetry boundary condition is used

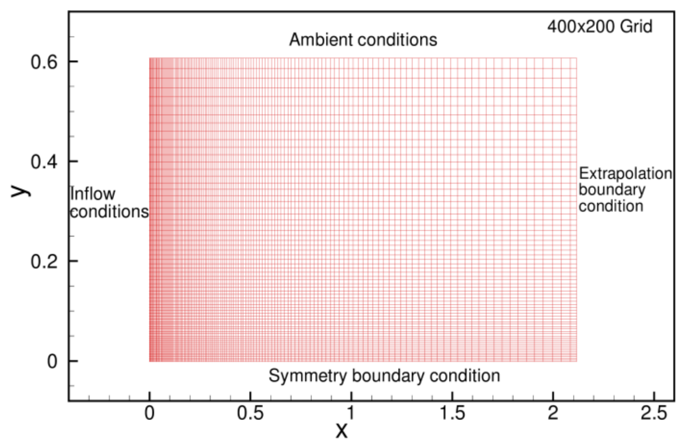

Figure 3: Grid structure and boundary condition details (Every 3 points in the x and y directions are skipped)

whereas, at the top boundary, the ambient conditions presented in Table **1** are directly imposed. At the exit boundary, the flow variables are extrapolated from the adjacent interior cells.

4.3 GRID CONVERGENCE

A grid refinement study is carried out for the Mach 3.13 jet with NPR = 17.2 exhausted into the still air. Three different grids are used: baseline grid with 400 × 200 points, a coarse grid with 200 × 100 points, and a fine grid with 600 × 300 points. The grid is stretched exponentially in both the x and y directions. Figure **4** plots the pressure along the jet centerline, normalized using the jet exit pressure. A good match is observed between the results predicted by the baseline (400 × 200) and the fine grid (600 × 300). We notice no significant difference in the flow structures in terms of their shape and positions computed using these two grids. Therefore, for all our computations (except for the cases with NPR = 50 and 100), the grid with 400 × 200 points is used. For the cases with NPR = 50 and 100, a larger domain in the x-direction (4 m) is required to capture the full flow-field, and for this, we have used a grid size of 500 × 200.

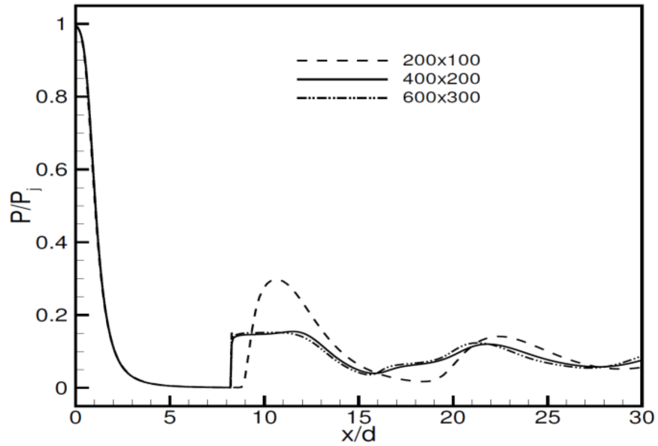

Figure 4: Variation of the normalized pressure along the jet centerline computed using three different grids. The parameters considered are: M_N = 3.13 and NPR = 17.2, and the jet is exhausted into the still air. Here, ‘d’ denotes the diameter of the nozzle.

5. RESULTS AND DISCUSSION

Figure **5** shows the variation in temperature in the computed flow field for the Mach 3.13 jet with NPR=17.2 and exhausted into the still air. The flow field features in terms of the shape of the jet, the expansion fan, the intercepting/barrel shock, the Mach disc, the reflected shock, and the slipstream, seen in an underexpanded supersonic jet flow-field at high NPR, are all reproduced clearly. A good qualitative match is observed with the results of Fu et al. [4] for the same jet conditions. To determine the effect of the change in NPR, jet Mach number, and ambient Mach number on the jet flow field, a parametric study is carried out using the inviscid axisymmetric computations for

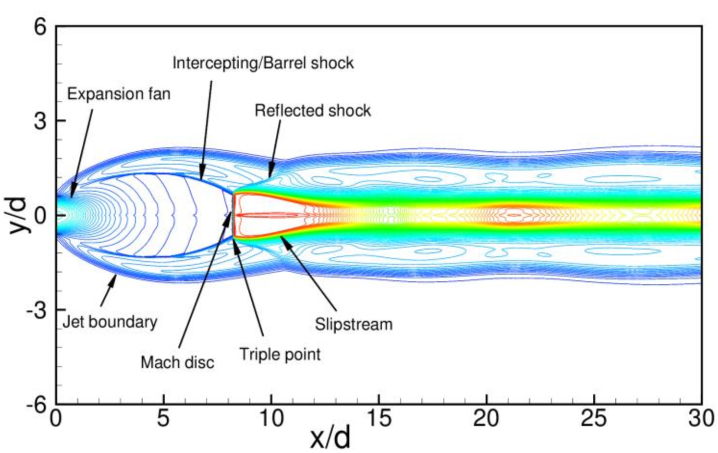

Figure 5: The flow-field for an underexpanded jet with Mach number of 3.13 and NPR = 17.2 is shown in terms of the temperature contours.

the jet conditions presented in Table **1** and is presented in the next subsections.

5.1 EFFECT OF NPR

A variation in NPR brings in the effect of a change in the altitude for an in-flight high-speed vehicle employing jet propulsion. The nozzle pressure ratio is varied by changing the ambient pressure, keeping the jet pressure constant. The increase in the NPR indicates an increase in the altitude and vice-versa. To study the sole effect of NPR on the jet structure, all other parameters, such as the nozzle radius and jet Mach number, are kept constant. A supersonic underexpanded jet with a Mach number of 3.13 is exhausted into still air, and five different values of NPR (4.3, 8.6, 17.2, 50 and 100) are considered for this study. Figure **6** plots the underexpanded jet flow-fields for different NPR values in terms of the Mach number contours. From Figs. **6a**-**6e** it can be observed that as the NPR increases, the length and size of the first shock-cell increases, due to an increase in the the jet expansion to match the reduced ambient pressure. An increase in the Prandtl-Meyer expansion angle with increase in the NPR leads to an increase in the initial inclination of the jet boundary with respect to the jet centerline, and the diameter of the jet. We notice a regular reflection of the incident oblique shock at low NPR (see Fig. **6a**), whereas Mach disc appears for higher NPRs. The height of the Mach disc also increases as the NPR is increased.

5.2 EFFECT OF JET MACH NUMBER

To study the effect of jet exit Mach number, we have considered three different jet Mach numbers 2, 3.13, and 4. Similar to the previous cases, underexpanded supersonic jet with an NPR = 17.2 is exhausted into the still air. The jet flow- fields shown in terms of the Mach number contours in Figs. **7a**-7**c** depict that, as the jet exit Mach number increases, the initial inclination of the jet boundary with respect to the jet centerline decreases. This can be attributed to the decrease in the Prandtl-Meyer angle with increase in the jet Mach number for the same NPR. We also observe that the length of the first shock-cell increases as the jet Mach number is increased. The distance from the jet exit to the maximum diameter also rapidly increases with the jet Mach number. Love and Grisby [6] mention that for the higher range of jet Mach numbers, an increase in the jet Mach number increases the range of jet static-pressure ratios in which no Mach disc occurs.

5.3 EFFECT OF AMBIENT MACH NUMBER

The effect of ambient Mach number on the underexpanded jet flow-field (NPR = 17.2) is studied by exhausting the Mach 3.13 jet into still air and to the supersonic air stream with Mach numbers 2 and 4. The jet flow-field is shown in terms of density contours in Fig. **8**. In Fig. **8a**, the jet is exhausted into the still air (M_∞ = 0), whereas, Figs. **8b** and **8c** depict two different jet

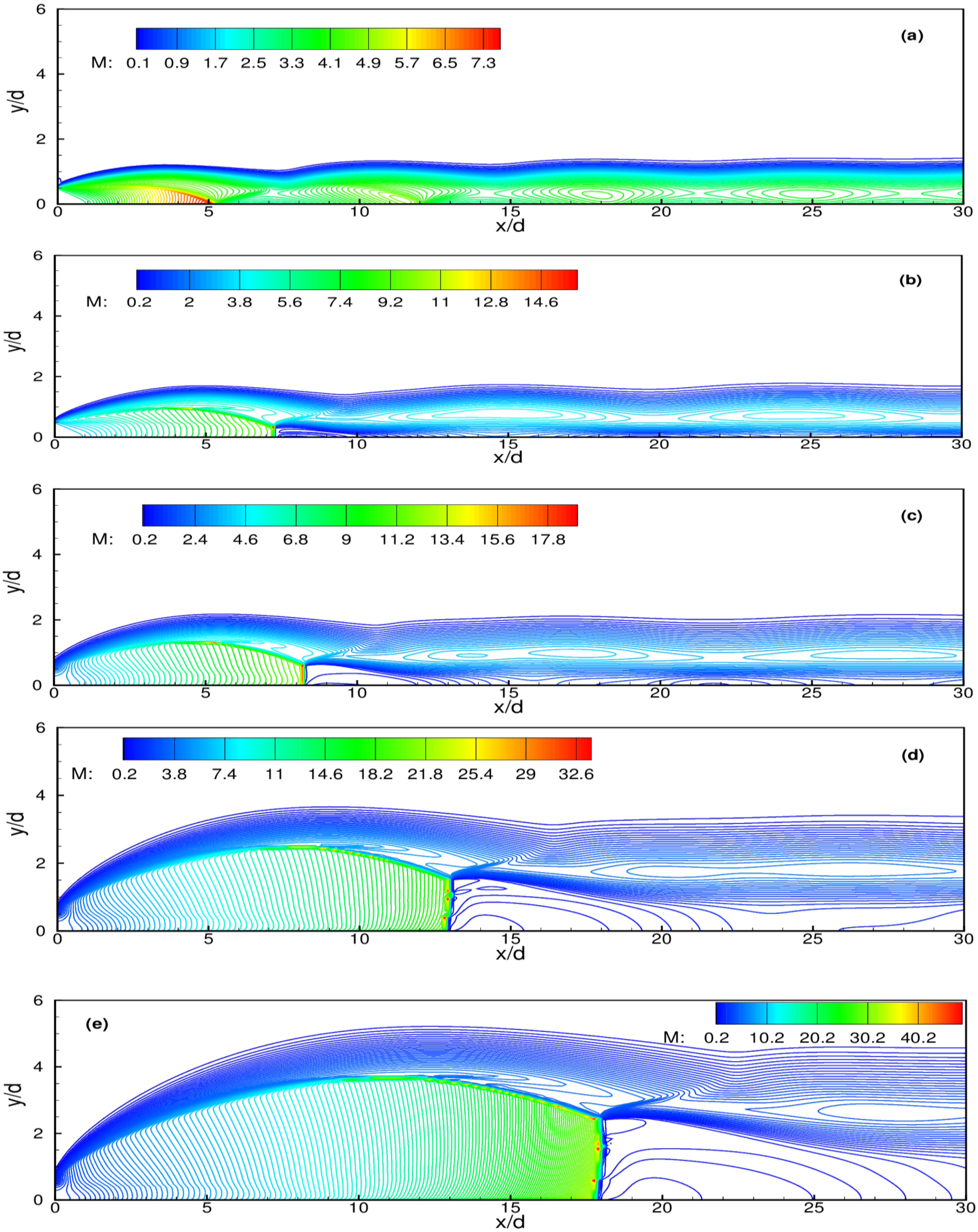

Figure 6: Effect of nozzle pressure ratio (NPR) on the jet flow-field at $M_N = 3.13$ and $M_\infty = 0$, shown in terms of Mach number contours. Five different NPR's considered are **(a)** 4.3; **(b)** 8.6; **(c)** 17.2; **(d)** 50 and **(e)** 100.

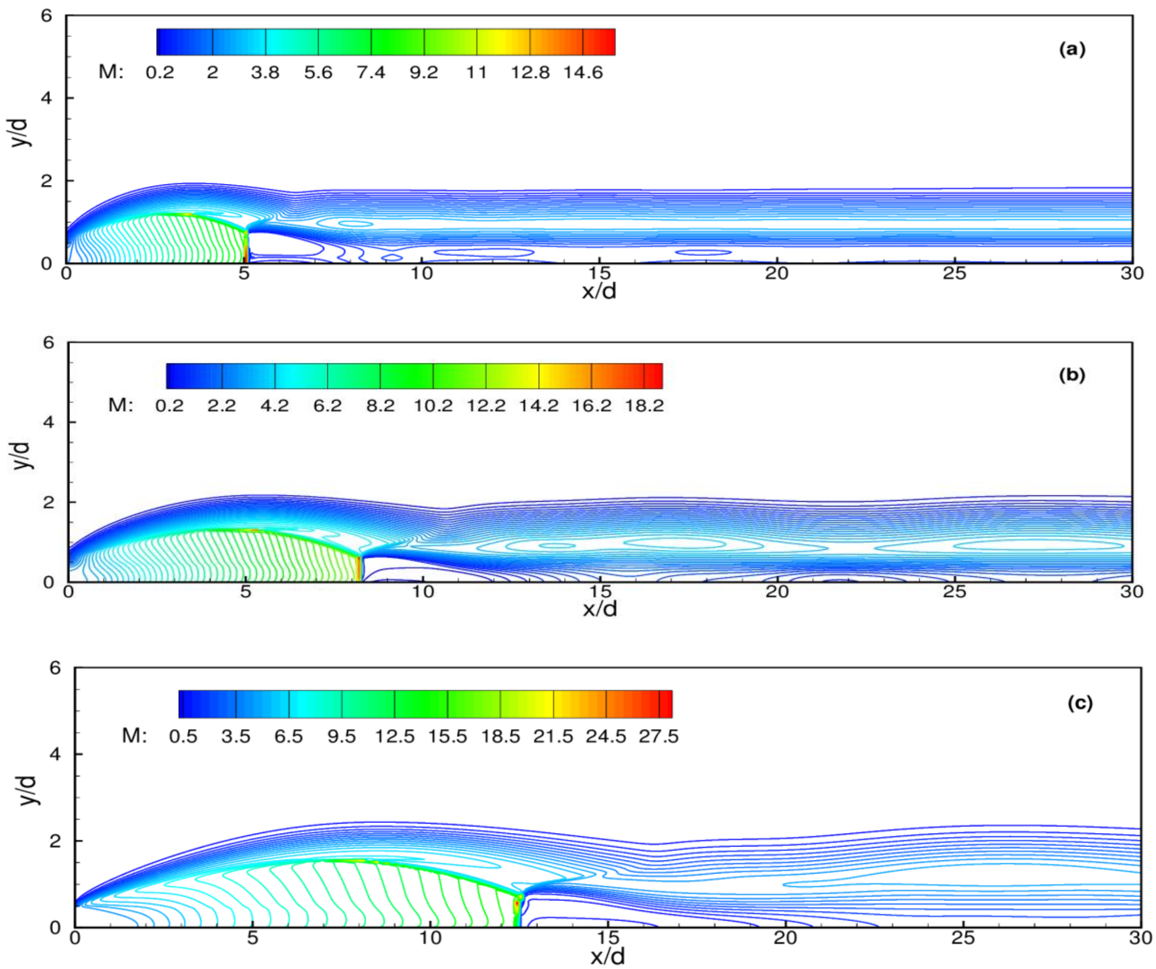

Figure 7: Effect of jet exit Mach number on the jet structure at NPR = 17.2 and the jet is allowed to exhaust into the still air. Three different jet Mach numbers considered are: **(a)** $M_N = 2$; **(b)** $M_N = 3.13$ and **(c)** $M_N = 4$.

flow-fields, where the jet is exhausted into a supersonic stream with Mach numbers $M_\infty = 2$ and 4, respectively. In contrast to the case with $M_\infty = 0$, the presence of supersonic external flow leads to the formation of oblique shocks to deflect the external flow over the expanded jet. We also observe a reduction in the initial inclination of the jet boundary for the case with jet exhausted into the supersonic flow. The increase in the ambient pressure downstream of the first external shock decreases the effective NPR, thereby reducing the extent of jet expansion. We also notice a significant increase in the first shock-cell length compared to the case with non-zero freestream Mach number. As opposed to the still air case, where a Mach disc is required to deflect the flow parallel to the jet centerline (see Fig. **8a)**, in the flow-field with supersonic Mach number, a regular reflection of the incident shock is enough for flow deflection (see Figs. **8b** and **8c)**. This is in line with previous studies where Love and Grisby [6] state that a supersonic external stream will permit a regular reflection to exist within the jet at much higher NPRs before giving way to a Mach reflection, which is not the case for the ambient still air.

The effect of increase in the ambient Mach number on the jet structure is investigated by comparing the $M_\infty = 2$ flow-field (Fig. **8b**) with another flow-field computed at $M_\infty = 4$ (Fig. **8c**). An increase in the ambient Mach number increases the strength of the first external shock and the shock becomes more inclined to the jet centerline. We also notice that the distance between the nozzle exit plane to the point of reflection of the first incident shock reduces with an increase in the M_∞ value.

CONCLUSION

A systematic parametric study of the underexpanded jets exhausted into the still air and supersonic streams is carried out. The parameters considered are the nozzle pressure ratio (NPR), nozzle exit Mach number, and the Mach number of the ambient air. Inviscid axisymmetric computations are performed using a

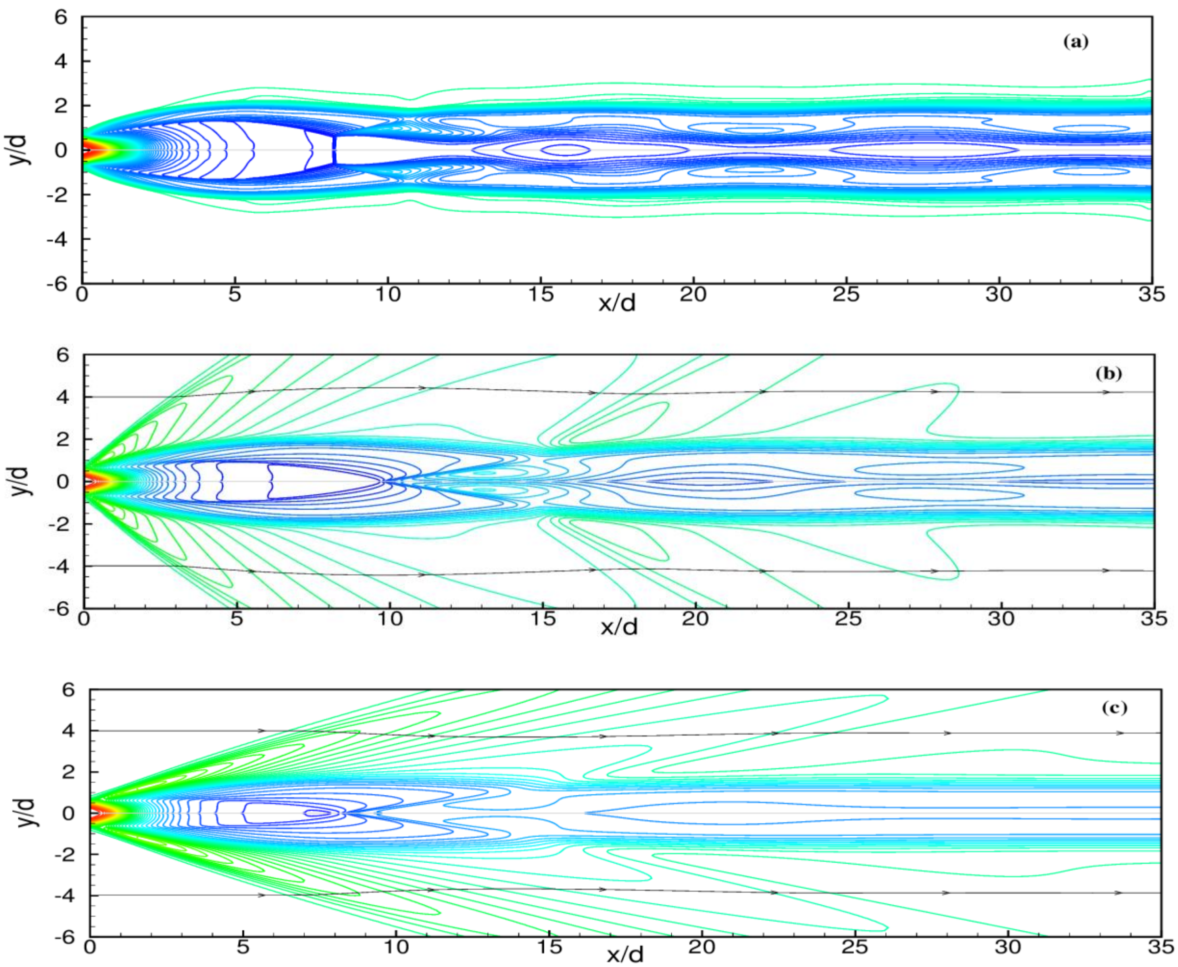

Figure 8: Effect of ambient Mach number on the jet structure at $M_N = 3.13$ and NPR = 17.2. Jets exhausted into **(a)** still air and into supersonic streams at **(b)** $M_\infty = 2$ and **(c)** $M_\infty = 4$ are shown using the density contours.

well-validated in-house RANS solver, and the results presented in this paper are grid-converged.

We notice a prominent increase in the first shock cell structure, along with an increase in the maximum jet diameter, with increase in the NPR value. It is observed that, keeping all other parameters constant, as the NPR is increased the regular reflection of the incident shock transitions to a Mach reflection.

The increase in the jet exit Mach number, with all other parameters constant, decreases the Prandtl-Meyer angle for which the same expansion is achieved. This results in a decrease in the initial jet boundary inclination. In addition, the length of the first shock-cell increases as the jet exit Mach number is increased.

As the underexpanded jet is allowed to exhaust into a supersonic stream, external shocks appear outside the jet boundary. The strength of these shocks increases as the Mach number of the ambient air increases. We note that the initial inclination of the jet boundary is less when the jet is exhausted into the supersonic stream compared to the case with ambient still air. The incident shocks have a regular reflection from the jet centerline as opposed to the presence of Mach reflection in the still air case.

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